where $x = -a/d$. The sum on the right is equal to

$$\sum_{s=0}^{m} (-1)^s \binom{m}{s} x^{m-s} \sum_{t=0}^{n} (-1)^t \binom{n}{t} t^s,$$

of which the inner sum vanishes for $s < n$, therefore for every s when $m < n$.

If now we set $k+1$ for n, k for a, -1 for d, and k for m, we have

$$\sum_{t=0}^{k+1} (-1)^t \binom{k+1}{t} (k-1)^k = 0,$$

that is,

$$\binom{k+1}{0} k^k - \binom{k+1}{1} (k-1)^k + \cdots + (-1)^{k-1} \binom{k+1}{k-1} 1^k + 0 - 1 = 0.$$

Also solved by S. A. Joffe.

GEOMETRY.

448. Proposed by S. W. REAVES, University of Oklahoma.

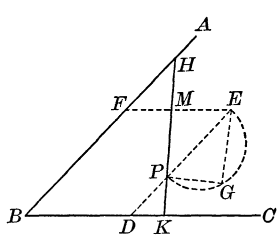
Through a given point P within a given angle to draw a line which shall form with the sides of the angle a triangle of a given area [Well's *New Plane Geometry* (1909), p. 153].

Solved in various ways by Horace Olson, Clifford N. Mills, George Y. Sosnow, Nathan Altshiller, and A. Holmes.

Solution by H. O. Hanson, East Elmhurst, N. Y.

Let $\angle ABC$ be the given angle, and P the given point within the angle.

Through P draw the line DE parallel to BA, and draw the line FE parallel to BC so that $BFED$ forms a parallelogram of the given area. On PE as hypothenuse construct the right triangle PEG so that $PG = PD$. Lay off $FH = GE$, and draw a line through H and P meeting BC at K and FE at M.



Then $\triangle BHK$ is the triangle required.

For, in the right triangle PEG, we have,

$$\overline{PG}^2 + \overline{GE}^2 = \overline{PE}^2,$$

or, since $PG = PD$, and $GE = FH$,

$$\overline{PD}^2 + \overline{FH}^2 = \overline{PE}^2.$$

Now, since the triangles PDK, FHM, and PEM are similar, and therefore proportional to the squares of their homologous sides, it follows that

$$\triangle PDK + \triangle FHM = \triangle PEM.$$

Adding the polygon $BFMPD$ to both sides of the above equivalence we get

$$\text{Area } BHK = \text{Area } BFED.$$

Remark. There will, in general, be two different solutions according as the side of the parallelogram drawn through P is taken parallel to BA or BC. If, however, the given area is such that $PE = PD$, the two solutions will be equal, and either solution will, in this case, give the *smallest triangle* that can be drawn with its side passing through P. If the given area is such that $PE < PD$, there will be no solution.

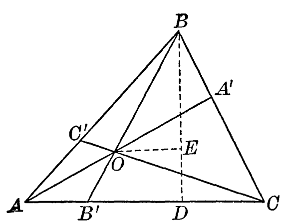
450. Proposed by W. L. WATSON, Moundsville, W. Va.

If three straight lines AA', BB', CC', drawn from the vertices of a triangle ABC to the opposite sides, pass through a common point O within the triangle, then

$$\frac{OA}{AA'} + \frac{OB}{BB'} + \frac{OC}{CC'} = 1.$$

SOLUTION BY MARCUS SKARSTEDT, Augustana College, Rock Island, Ill.

Draw BD perpendicular to AC, and OE perpendicular to DB. Then



$$\frac{OB'}{BB'} = \frac{ED}{BD} = \frac{\triangle AOC}{\triangle ABC}.$$

Similarly,

$$\frac{OA'}{AA'} = \frac{\triangle COB}{\triangle ABC} \quad \text{and} \quad \frac{OC'}{CC'} = \frac{\triangle ABO}{\triangle ABC}.$$

Adding, we get

$$\frac{OA'}{AA'} + \frac{OB'}{BB'} + \frac{OC'}{CC'} = \frac{\triangle COB + \triangle AOC + \triangle ABO}{\triangle ABC} = 1.$$

Solved similarly by A. M. HARDING, NATHAN ALTSHILLER, R. M. MATHEWS, A. H. HOLMES, T. DANTZIG, PAUL CAPRON, E. E. WHITFORD, HORACE OLSON, CLIFFORD N. MILLS, A. L. MCCARTY, and GEORGE Y. SOSNOW.

451. Proposed by CLIFFORD N. MILLS, So. Dakota State College.

Determine the sides of an isosceles triangle of given area, having given that the sum of its sides is equal to the sum of its base and altitude.